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Characterization of resolution and uniqueness in crosswell direct-arrival traveltime tomography using the Fourier projection slice theorem

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ABSTRACT

The process of acquiring a crosswell seismic direct-arrival traveltime data set can be approximated by a series of truncated plane-wave projections through an interwell slowness field. Using this approximation, the resolution and uniqueness of crosswell direct-arrival traveltime tomograms can be characterized by invoking the Fourier projection slice theorem, which states that a plane-wave projection through an object constitutes a slice of the object's spatial spectrum.

The limited vertical aperture of a crosswell survey introduces a small amount of nonuniqueness into the reconstructed tomogram by truncating the plane-wave projection. By contrast, the limitations on angular aperture have a significant effect on resolution. The reconstructed tomogram is smeared primarily along the limiting projection angles, with the amount of smearing dependent upon the well spacing and the angular aperture. The amount of smearing was found to be inversely proportional to tan $\Delta\phi$, where $\Delta\phi$ is the angular aperture illuminating a sector of the interwell plane. Consequently, the amount of smearing can be large where the angular aperture becomes small, such as at the top and bottom of the tomogram. For interwell sectors illuminated by large angular apertures, Fresnel zone effects will generally be the limiting factor in crosswell tomogram resolution. However, in some circumstances, angular aperture effects may control the tomogram resolution.

The effects of angular aperture and direct-arrival Fresnel zones produce tomograms with spatial resolution that is dependent upon the well spacing. This study indicates that direct-arrival traveltime tomography will not usually produce tomograms with substantially greater resolution than surface seismic techniques for normal oil and gas well spacings.

INTRODUCTION

In the past ten years, there has been an extensive amount of research and field study related to direct-arrival traveltime tomography for crosswell geometries (Justice et al. 1989, Bregman et al. 1989a). It has been known for many years (Mersereau and Oppenheim, 1974) that any band-limited object can be reconstructed perfectly from projections, provided that the projections illuminate the object from all angles. Projection reconstruction imaging is responsible for the development of medical tomography methods such as the CAT scan and has motivated much of the initial research into crosswell tomography for reservoir monitoring and reservoir characterization.

For the typical crosswell geometry (with vertical boreholes), direct-arrival traveltime tomography produces interwell velocity reconstructions (tomograms) that must contend with a limited aperture of raypath angles that traverse a particular sector of the interwell plane. In simple terms, limited angular aperture means that certain frequencies of the interwell 2-D spatial spectrum are not illuminated (Hardage, 1992). Although most recent discussions of traveltime tomography resolution have dealt with the Fresnel zone effects caused by the limited temporal bandwidth of the seismic data (e.g., Schuster and Quintus-Bosz, 1993), several investigators such as Menke (1984) and Bregman et al. (1989) have investigated the effects of limited angular aperture on the resolution and uniqueness of direct-arrival traveltime tomography. Menke (1984) found that the resolution was dependent upon the angular survey aperture and that for most crosswell survey geometries, horizontal resolution was less than vertical resolution. In addition to the effect of

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angular aperture on resolution, Bregman et al. (1989b) found that the limited aperture of the crosswell geometry created large-scale velocity ambiguities in the crosswell tomogram.

In this paper, we use the Fourier projection slice theorem to separate the effects of limited angular aperture from the effects of what we will term projection truncation, or effects introduced by bounded source and receiver positions, in crosswell direct-arrival traveltime tomography. We find that angular aperture affects resolution, and projection truncation influences the variation in the reconstructed interwell image. We quantify the constraints that angular aperture limitations place on the potential resolution that can be obtained with crosswell direct-arrival traveltime tomography. We find that the resolution depends on angular aperture and well spacing, and that the resolution changes for different locations within the interwell plane.

THE FOURIER PROJECTION SLICE THEOREM AND THE CROSSWELL GEOMETRY

The process of transmitting a plane wave through a 2-D object, $g\,(x,\,z)$, can be described by the projection operator $p_\phi(R)$ where

$$p_\phi(R) = \int_\varepsilon g(R\cos\phi - \varepsilon\sin\phi,\; R\sin\phi + \varepsilon\cos\phi)\,d\varepsilon. \qquad (1)$$

R is the axis onto which the projection is made, ε is the axis perpendicular to R, and ϕ is the angle that the R-axis makes with the z-axis.

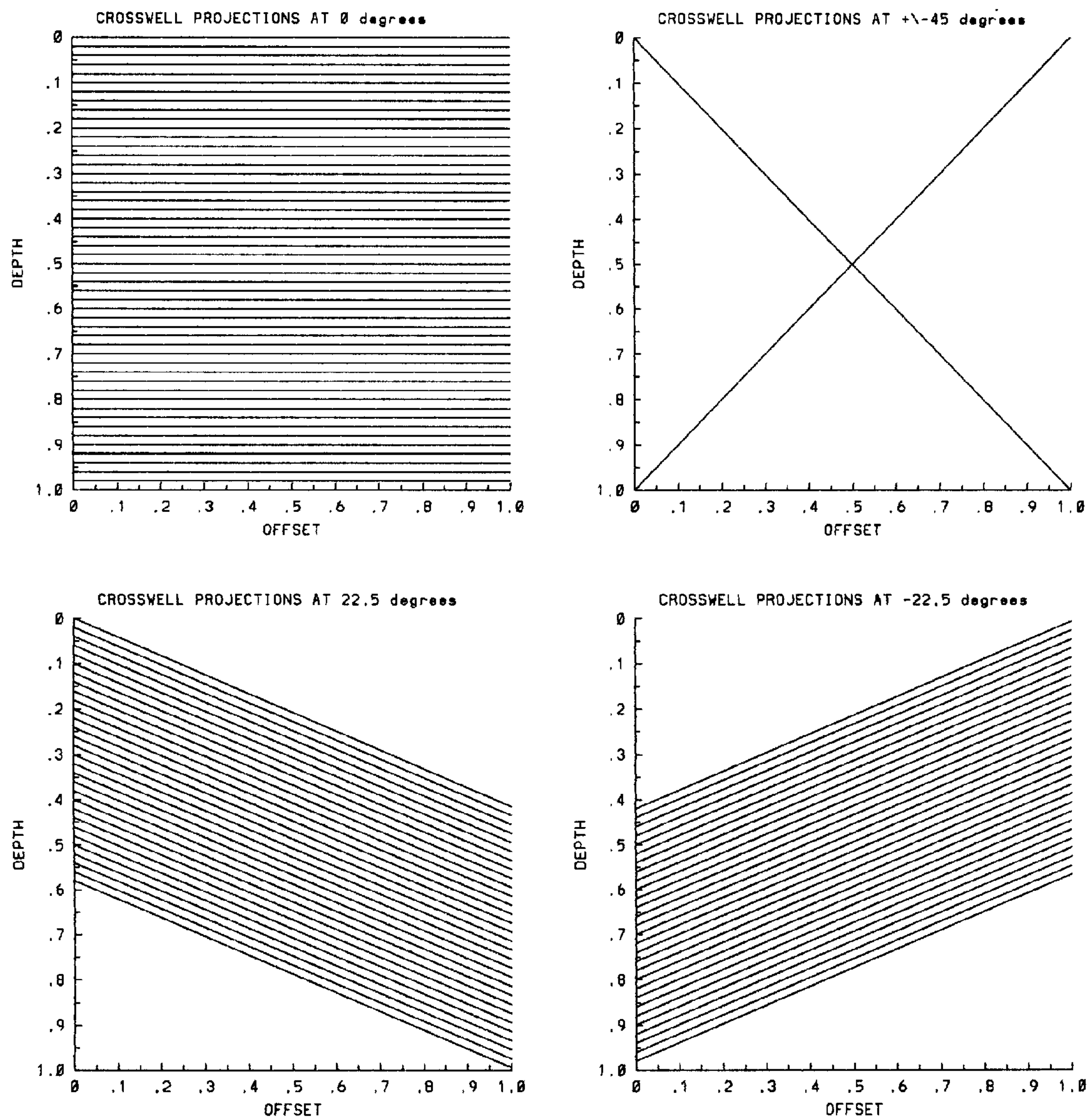

FIG. 1. Crosswell direct-arrival raypaths sorted as a series of projections.

Let $P_\phi(\rho)$ be the 1-D Fourier transform of the projection operator and $G(k_x, k_z)$ be the 2-D Fourier transform of the object, where k_x is the horizontal wavenumber and k_z is the vertical wavenumber. Then the Fourier projection slice theorem (Mersereau and Oppenheim, 1974) states that

$$P_\phi(\rho) = G(\rho \cos \phi, \rho \sin \phi). \tag{2}$$

In other words, the Fourier transform of the projection operator constitutes a "slice" of the 2-D Fourier transform of the object, where the "slice" makes an angle ϕ with respect to the k_x axis. By taking many such slices (i.e., projections) at small angular increments through the object, a representation of the 2-D Fourier transform of the object can be constructed. Provided the object spectrum is band-

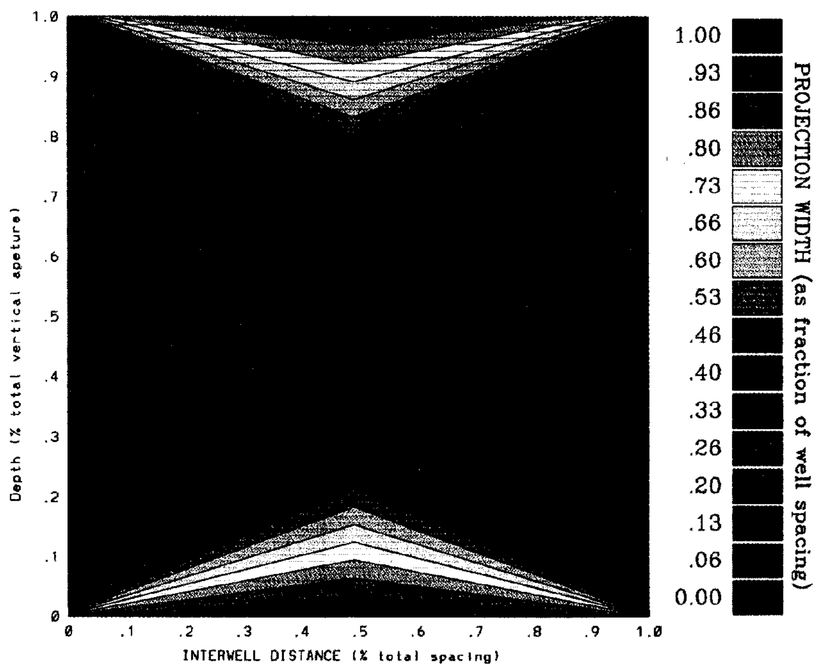

FIG. 2. Minimum projection width as a function of position within the interwell region.

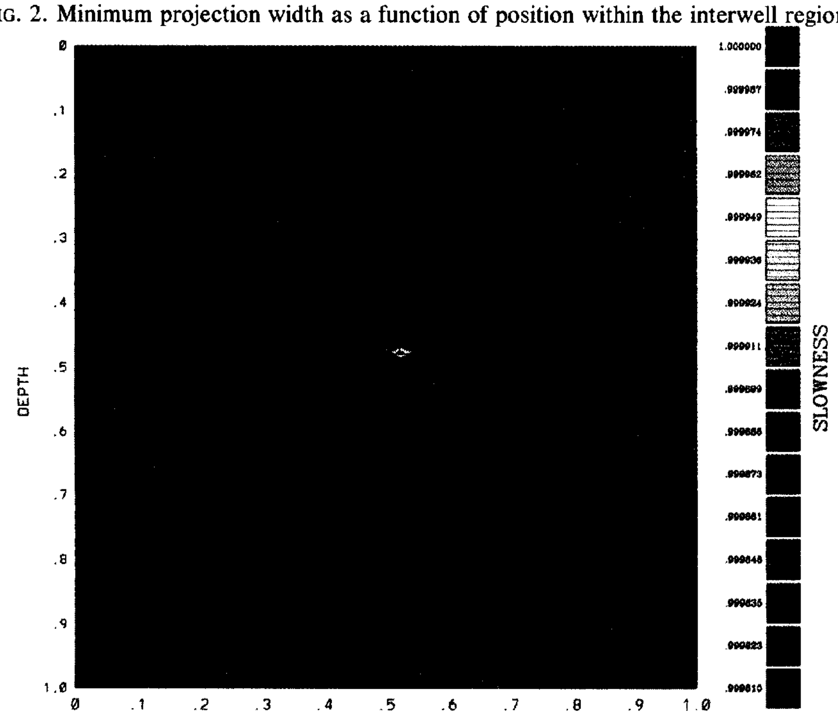

FIG. 3. Back-projected slowness image from a constant velocity model, where the constant velocity is equal to one.

limited, the object can be reconstructed perfectly from the projections through the process of "back-projection" (Worthington, 1984).

The direct-arrival traveltimes from multiple source-receiver pairs in a crosswell survey can be thought of as a truncated projection of a plane wave onto the interwell slowness field. Each common offset gather (where offset is defined as the vertical source-receiver separation) constitutes a truncated projection, where the limits of the projection are dependent on the minimum and maximum source and receiver positions and the source-receiver offset. Consequently, as shown in Figure 1, any point in the interwell region can be thought of as being illuminated by a series of truncated projections, with the range of projection angles dictated by the angular aperture of the survey and the location of the point within the interwell region.

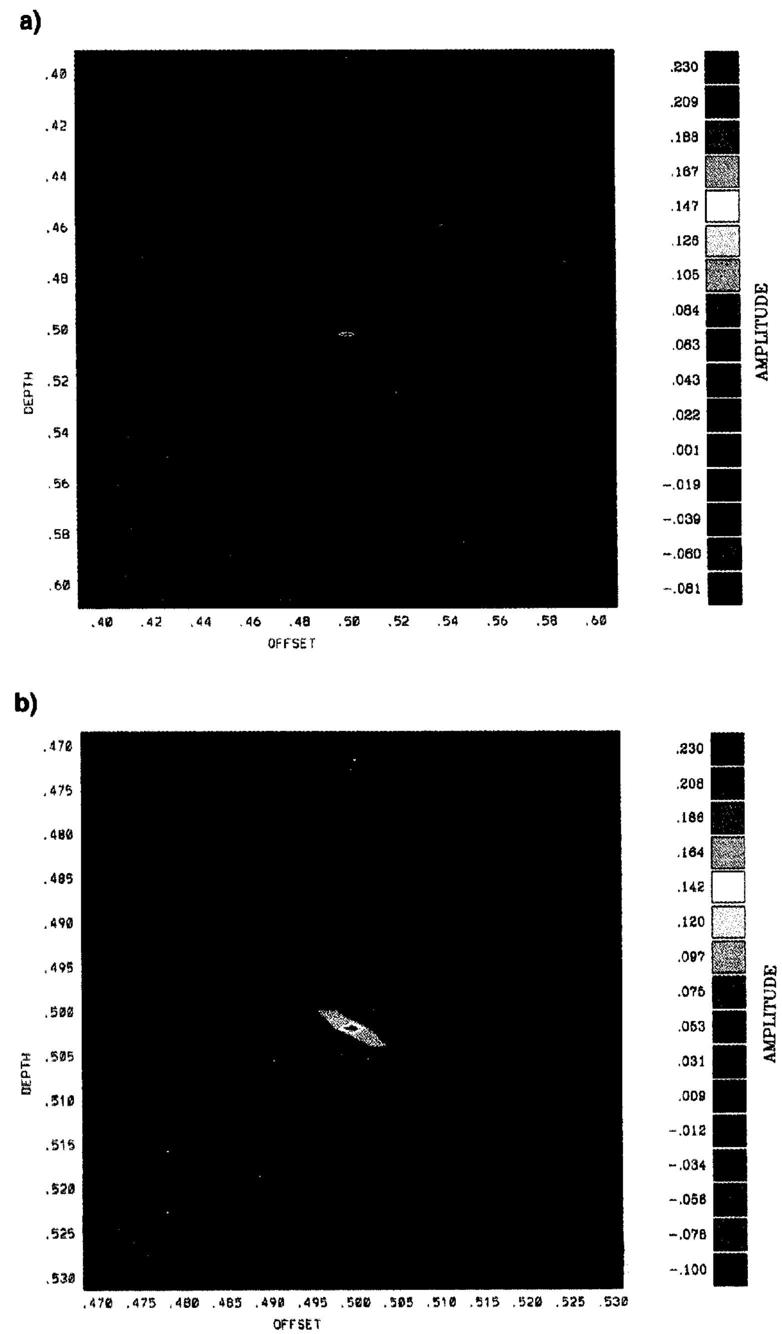

FIG. 4. 2-D spatial impulse response of several projection filters representing selected angular apertures: (a) $\phi_{min} = -45$, $\phi_{max} = 45$; (b) $\phi_{max} = 0$, $\phi_{min} = -45$; (c) $\phi_{min} = -1$, $\phi_{max} = 1$. The vertical and horizontal axes are plotted as a fraction of vertical aperture and well spacing, respectively. The initial impulse had unit amplitude.

EFFECTS OF PROJECTION TRUNCATION

To characterize the effects of truncated projections, we consider a uniform crosswell survey between two vertical boreholes. Sources and receivers are assumed to be deployed continuously over a length H of the source and receiver boreholes. The well separation is W. A single truncated projection, $p_{\phi\text{trunc}}(R)$, can be written in terms of the untruncated projection, $p_{\phi}(R)$, as

$$p_{\phi\text{trunc}}(R) = p_{\phi}(R)\ \text{Rect}\ (R/\Delta R), \qquad (3)$$

where

$$\Delta R = H \cos \phi - W \sin |\phi|,\ (\tan \phi < H/W), \qquad (4)$$

and Rect (R) is the rectangle or boxcar function defined in Bracewell (1982),

$$\text{Rect}\ (R) = 1 \text{ if } |R| < 0.5$$

$$\text{Rect}\ (R) = 0.5 \text{ if } |R| = 0.5$$

$$\text{Rect}\ (R) = 0 \text{ if } |R| > 0.5.$$

Taking the 1-D Fourier transform of equation (3) we obtain:

$$P_{\phi\text{trunc}}(\rho) = \Delta R\ \text{sinc}\ (\Delta R \rho) * P_{\phi}(\rho), \qquad (5)$$

where sinc (R) = sin $\pi R/\pi R$ and $*$ denotes convolution. Invoking the projection slice theorem, we can see that for an individual projection, truncation leaks energy into different ρ values. In terms of the 2-D spectrum of the interwell region, the truncated projection leaks spectral energy along the line $k_z = k_x \tan \phi$. As in conventional signal processing applications, the leakage can be reduced through techniques such as tapered windowing. However, some amount of spectral leakage will always be present, particularly when the projection width goes to zero. Figure 2 shows the minimum projection width as a function of position within the interwell region, assuming continuous sampling and a 1:1 survey geometry. The zones with a small projection width will have the most spectral leakage and therefore the greatest ambiguity in the reconstructed image. Consequently, projection truncation can be interpreted as a smoothing or whitening filter applied to the 2-D spectrum, which increases the variation of the reconstructed 2-D image, but does not affect resolution.

We can evaluate the effects of projection truncation on the reconstructed image by examining the back projection of a constant slowness model, $g(x, z) = 1$. The projection of a constant background object is again a constant, and the 1-D Fourier transform of the truncated projection operator is:

$$P_{\phi\text{trunc}}(\rho) = \Delta R\ \text{sinc}\ (\Delta R \rho). \qquad (6)$$

Figure 3 shows the inverted slowness tomogram resulting from truncated projections through a constant slowness model. The velocity values have the most variation in the center of the image, where the minimum projection widths intersect. The magnitude of the deviation from the constant slowness initial model is quite small (less than .01 percent), suggesting that projection truncation does not significantly affect the accuracy of the velocities obtained with direct-arrival traveltime tomography.

EFFECTS OF LIMITED ANGULAR APERTURE ON RESOLUTION

It is well known that the limited angular aperture in direct-arrival raypaths leads to a decrease in resolution of the tomographically reconstructed interwell image (Menke, 1984). Using the Fourier projection slice theorem, we can quantify the resolution in terms of the spatial impulse response, also known as the point-spread function, of a limited aperture (but untruncated) projection operator. For the crosswell geometry shown in Figure 2, the angular dependence of the projection operator for any point in the interwell image can be written as:

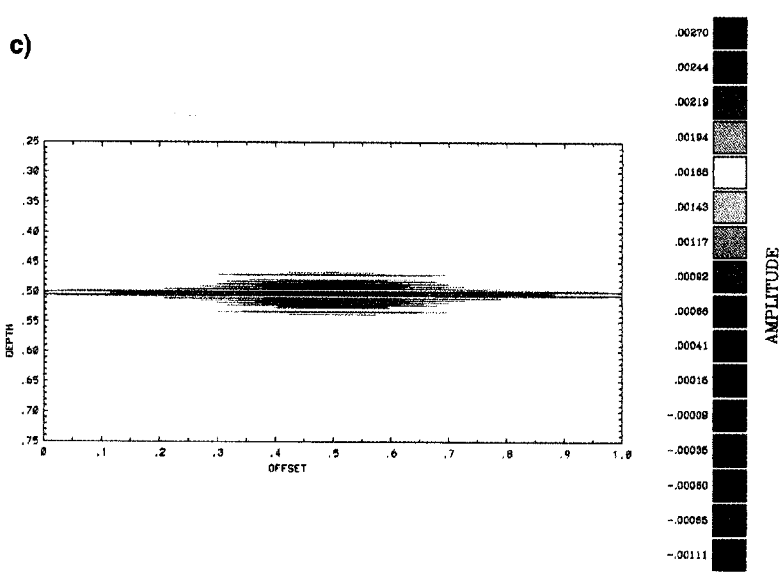

FIG. 4. (continued)

$$P_\rho[\phi(x, z)] = \text{Rect}\,[\phi/(\phi_{max} - \phi_{min})], \qquad (7)$$

where ϕ_{max} and ϕ_{min} are the maximum upward and downward projection angles for a particular point (x, z) in the interwell region. Figure 4 shows the 2-D spatial impulse response of the angle-limited projection operator, $P_\rho[\phi(x, z)]$, for selected angular apertures. These impulse responses were produced by digitally inverse transforming the pie-slice velocity filter that defines the range of projection angles illuminating a point in the interwell region (Hardage, 1992). When the pie-slice filter is untapered [i.e., formulated like equation (7)], the impulse is smeared primarily along the slopes that correspond to the edges (ϕ_{max} and ϕ_{min}) of the filter in the transformed domain. In other words, the resolution is lowest in the directions that correspond to the edges of the filter. The 2-D impulse response can be modified by tapering the 2-D projection filter to reduce the edge effects. Tapering essentially amounts to weighting the raypath angles–deemphasizing the outlying angles. From window design theory, it is well known that with tapering, the smearing of the impulse response at the limiting angles will be reduced, but the smearing at intermediate angles will be increased. In other words, with tapering, the main lobe width is increased while the side lobes are reduced. Figure 4 also shows that the amount of smearing increases as the angular aperture decreases.

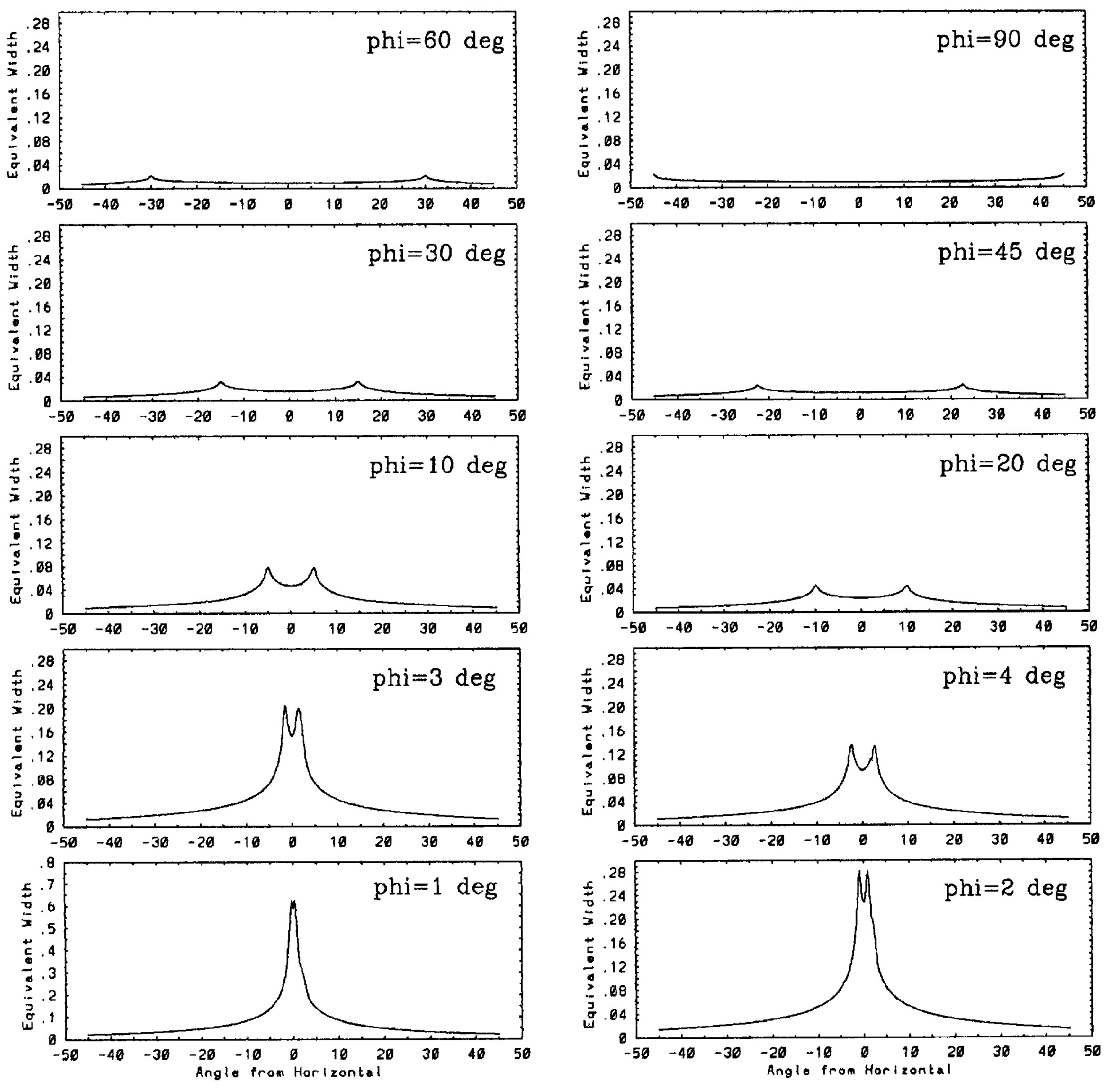

FIG. 5. Equivalent width versus angle (plotted as a fraction of the well spacing) of selected angular apertures (phi) centered about a horizontal projection.

The magnitude of the smearing as a function of angular aperture can be estimated by computing the equivalent width of the impulse response along different angles that intersect the center of the image. The equivalent width of a function is the width of a rectangle with the same area as the function (Bracewell, 1982). For our case, we can define the equivalent width, E_ϕ as

$$E_\phi = \int_L |f(x, z)|/f(0, 0). \tag{8}$$

where $f(x, z)$ is the impulse response of the angle-limited projection operator, and L is a line that intersects the center of the impulse response at an angle ϕ. Figure 5 shows E_ϕ as a fraction of the well spacing for several angular apertures centered about $\phi = 0$. Note that the smearing increases as the angular aperture decreases.

We can also characterize the angle-limited projection operator by examining the analytic representation of its impulse response. In the 2-D spatial frequency domain (k_x, k_z), the angle-limited projection operator can be written as Rect $[k_z \tan(\Delta\phi)/k_x]$, where Rect[ ] is defined above. The inverse 2-D transform $Imp(x, z)$ is then related to the angular aperture $\Delta\phi$ through

$$Imp(x, z) \sim \delta[z \tan(\Delta\phi)/x]/z, \tag{9}$$

where δ is a 2-D blade function defined in Bracewell (1982). The expression given in equation (9) confirms the aperture dependence of the projection operator shown in Figure 5. As the aperture increases, the influence of the $1/z$ term in equation (9) becomes more pronounced because the 2-D blade functions become more vertically oriented.

By combining the analytic representation shown in equation 9, and the numerical results shown in Figure 5, we derived an empirical relationship between the equivalent width at the limiting projection angles $E_{\phi=\phi_{max}}$ the angular aperture, $\Delta\phi$, and the well spacing, W:

$$E_{\phi=\phi_{max}} = W/58.8 \tan(\Delta\phi). \tag{10}$$

This expression has a correlation coefficient of 0.997 with the maximum equivalent widths shown in Figure 5. Equation (10) is a measure of the spatial resolution of crosswell traveltime tomography. Impulsive heterogeneities will be smeared, primarily at the limiting projection angles, over a distance with an equivalent width $E_{\phi=\phi_{max}}$. Equation (10) reemphasizes the necessity of having sufficient angular aperture at all sectors of interest in the interwell plane. Interestingly, it is not essential to have an extremely wide aperture [consistent with the results of Menke (1984)], but it is extremely important that the aperture be larger than a few degrees everywhere. In cases with sparse ray sampling or severe ray focusing (into, for example, high velocity layers), the angular aperture may become very small in some sectors of the interwell region. In these instances, spatial resolution will be lowered dramatically. Figure 6 shows the angular aperture as a function of position within the interwell region. Except for an ellipsoidal zone at the very top and bottom of the image, the reconstructed image should be well resolved because the angular aperture is larger than a few degrees.

Another parameter that has been used to describe the resolution of crosswell direct-arrival traveltime tomography is the Fresnel zone of the direct arrival, approximated in Williamson and Worthington (1993) as $(\lambda R)^{1/2}$, where R is the path length of the direct arrival and λ is the wavelength

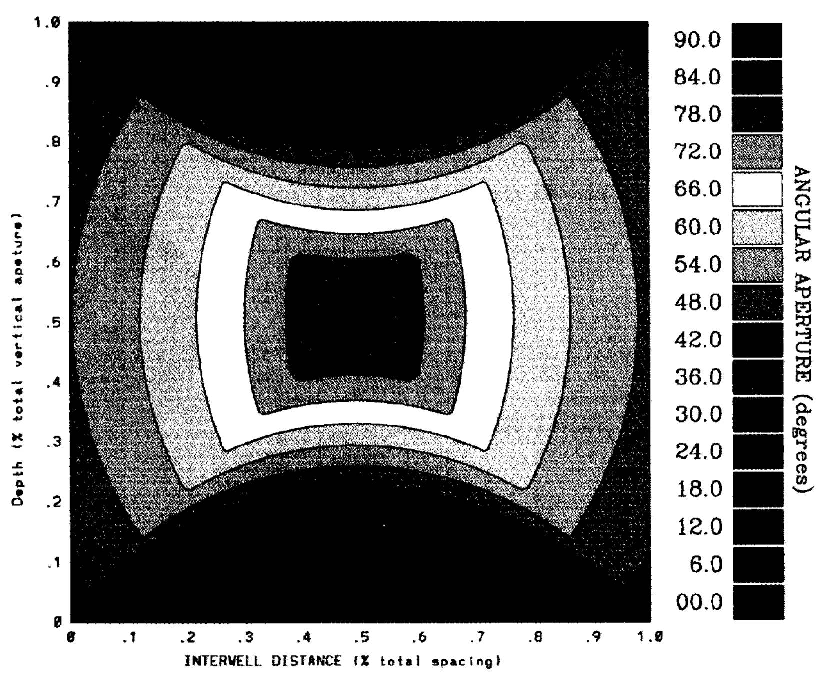

FIG. 6. Angular aperture as a function of position.

of the direct arrival. In reality, the Fresnel zone is also spatially variable, being smallest near the source and receiver wells and largest midway between the wells (Vasco and Majer, 1993). Equating equation (10) with the analytic representation of the Fresnel zone and assuming a horizontal raypath [so that W can be replaced by R in equation (10)], we find that the aperture-related resolution is approximately equal to the Fresnel zone when

$$\lambda/R = [58.8 \tan(\Delta\phi)]^{-2}. \qquad (11)$$

For crosswell seismic data where the travelpath length is around fifty wavelengths, the Fresnel zone is the limiting factor dictating crosswell spatial resolution. However, when the angular aperture becomes very small (less than about 10 degrees) or when the Fresnel zone is not assumed to limit the resolution of traveltime tomography (for example, if first breaks can be accurately estimated), the angular aperture can be the limiting factor that controls spatial resolution of crosswell traveltime tomography.

If we assume that the Fresnel zone limits the spatial resolution that can be obtained with crosswell direct-arrival traveltime tomography, then we must record data with a center frequency of about 2000 Hz to obtain Fresnel zones at normal oil and gas well spacings (about 600 m) that are equal to typical surface seismic resolution (~20 m). If we wish to obtain significantly higher resolution, we need even higher frequencies. It is probably unrealistic to expect that these frequencies can be propagated over such distances. Even if the Fresnel zone does not limit the spatial resolution, we would need angular apertures that are greater than about 45 degrees everywhere in the interwell region to obtain spatial resolution that is significantly better than surface seismic resolution. For long well spacings in particular, these angular apertures may be difficult, if not impossible to obtain, particularly at the base of the image.

CONCLUSIONS

Using the Fourier projection slice theorem, crosswell tomography using direct arrivals can be formulated as a series of truncated projections onto the interwell slowness field over a limited range of projection angles. The effect of projection truncation is to introduce nonuniqueness into the solution through wavenumber smearing of the information. This effect is identical to spectral leakage effects created when a frequency spectrum is computed from a windowed time series. The spectral leakage is greatest for those projections with the smallest width. Limited angular aperture acts like a pie-slice velocity filter on the 2-D spatial spectrum of an interwell heterogeneity. The impulse response of the pie-slice velocity filter indicates the magnitude and direction of information smearing in a tomographically-reconstructed interwell image. For equally weighted angular projections, information is primarily smeared along the limiting projection angles.

From this study, we found that the nonuniqueness introduced by projection truncation was very small, and that the aperture-related resolution was inversely proportional to angular aperture and well spacing. We also showed that for zones of the interwell region covered by a large angular aperture, the Fresnel zone was the limiting factor in determining resolution. However, the angular aperture can become very small at the edges of the survey in zones with sparse ray coverage or in zones with ray focusing. The information from these zones will be smeared in the reconstructed tomogram. The effects of limited angular aperture and limited projection aperture should be considered when designing crosswell surveys and interpreting crosswell direct-arrival traveltime tomograms. When combined, the Fresnel zone effect and the angular aperture effect indicate that direct-arrival traveltime tomography will *not*, in general, provide higher spatial resolution than surface reflection surveys at normal (40 to 160 acre) oil and gas well spacings. Crosswell imaging techniques that use the full waveform such as crosswell reflection imaging or crosswell diffraction tomography will be required to obtain resolution that is significantly better than surface seismic data.

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